

Vector Boson Production at Hadron Colliders: Results from HERWIG and Resummed Calculations¹

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Abstract. We discuss vector boson production at hadron colliders and the treatment of the initial-state radiation according to Monte Carlo parton shower simulations and resummed calculations. In particular, we investigate the effect of matrix-element corrections to the HERWIG event generator on W/Z transverse momentum distributions.

INTRODUCTION

The production of vector bosons W , Z and γ [1] is one of the most interesting processes in the phenomenology of hadron collisions and provides an environment to test both Quantum Chromodynamics and the Standard Model of electroweak interactions (see [2,3] for a review). The lowest-order processes $q\bar{q}' \rightarrow V$ are not sufficient to make reliable predictions, but the initial-state radiation should be taken into account. Monte Carlo event generators and resummed analytical calculations are available tools to describe the multiple radiation accompanying the incoming hadrons.

Standard Monte Carlo algorithms [4,5] describe the initial-state parton showers in the soft/collinear approximation, but can have ‘dead zones’, where no radiation is permitted. The radiation in these regions can be generated by the use of the exact first-order matrix element. Referring to the HERWIG event generator, matrix-element corrections to Drell–Yan processes have been implemented in [6], following the general method of [7], and included in the new version HERWIG 6.1 [8].

Another possible approach consists of performing an analytical resummation of the large logarithmic coefficients which multiply the strong coupling constant. Considering the transverse momentum q_T distribution, logarithms of the ratio m_V/q_T ,

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m_V being the vector boson mass, arise in calculating higher-order corrections to the Born process. The resummation of these logarithms, which are large at small q_T , was initially proposed by Dokshitzer, Dyakonov and Troyan (DDT) [9], then accomplished by Collins, Stermann and Soper (CSS) [10]. CSS performed the resummation in the space of the impact parameter b , which is the Fourier conjugate of q_T . Their results have been implemented numerically in [11,12], while more recent analyses can be found in [13–15], where the resummation is performed in both q_T - and b -space.

In this paper, we review some results for the W/Z transverse momentum distribution according to the HERWIG event generator and resummed calculations.²

THE HERWIG PARTON SHOWER ALGORITHM

HERWIG simulates the initial-state radiation in hadron collisions according to a ‘backward evolution’ [17], in which the scale is reduced away from the hard vertex and traces the hard-scattering partons back into the incoming hadrons. The branching algorithm relies on the universal structure of the elementary probability in the leading infrared approximation. The probability of the emission of an additional soft/collinear parton from a parton i is given by the general result:

$$dP = \frac{dq_i^2}{q_i^2} \frac{\alpha_S \left(\frac{1-z_i}{z_i} q_i \right)}{2\pi} P_{ab}(z_i) dz_i \frac{\Delta_{S,a}(q_{i\max}^2, q_c^2)}{\Delta_{S,a}(q_i^2, q_c^2)} \frac{x_i/z_i}{x_i} \frac{f_b(x_i/z_i, q_i^2)}{f_a(x_i, q_i^2)}. \quad (1)$$

The ordering variable is $q_i^2 = E^2 \xi_i$, where E is the energy of the parton that splits and $\xi_i = \frac{p \cdot p_i}{E E_i}$, with p and p_i being the four-momenta of the splitting and of the emitted parton respectively; z_i is the energy fraction of the outgoing space-like parton with respect to the incoming one; $P_{ab}(z)$ is the Altarelli–Parisi splitting function for a parton a evolving in b . In the approximation of massless partons, we have $\xi_i = 1 - \cos \theta$, where θ is the emission angle to the incoming hadron direction. For soft emission ($E_i \ll E$), ordering according to q_i^2 corresponds to angular ordering. When the emission is hard, the energy of the radiated parton is similar to that of the splitting parton, so q_i^2 -ordering corresponds to transverse momentum ordering. In (1) $f_a(x_i, q_i^2)$ is the parton distribution function for the partons of type a in the initial-state hadron, x_i being the parton energy fraction. The function

$$\Delta_S(q_2^2, q_1^2) = \exp \left[-\frac{\alpha_S}{2\pi} \int_{q_1^2}^{q_2^2} \frac{dk^2}{k^2} \int_{Q_1/Q_2}^{1-Q_1/Q_2} dz P(z) \right] \quad (2)$$

is the Sudakov form factor, expressing the probability of no-resolvable branching in the range $q_1^2 < q^2 < q_2^2$. The ratio of form factors in (1) is therefore the probability of no branching at higher values of q_i^2 . Unitarity dictates that the Sudakov form

²⁾ See also [16] for a similar comparison for Higgs production at hadron colliders.

factor sums up all-order virtual and unresolved contributions. In (1), q_{imax} is the maximum value of q , fixed by the hard process, and q_c is the value at which the backward evolution is terminated, corresponding, in the case of HERWIG, to a cutoff on the transverse momentum of the showering partons. However, since q_c is smaller than the minimum scale at which the parton distribution functions are evaluated, an additional cutoff on the evolution variable q_i^2 has to be set.

If the backward evolution has not resulted in a valence quark, an additional non-perturbative parton emission is generated to evolve back to a valence quark. Such a valence quark has a Gaussian distribution with respect to the non-perturbative intrinsic transverse momentum in the hadron, with a width $q_{T\text{int}}$ that is an adjustable parameter and whose default value is zero.

We need finally to specify the showering frame, the variables q_i^2 and z_i being frame-dependent. One can show that, as a result of the q^2 -ordering, the maximum q -values of two colour connected partons i and j are related via $q_{\text{imax}}q_{j\text{max}} = p_i \cdot p_j$, which is Lorentz-invariant. For vector boson production, symmetric limits are set in HERWIG: $q_{\text{imax}} = q_{j\text{max}} = \sqrt{p_i \cdot p_j}$. Furthermore, the energy of the parton which initiates the cascade is given by $E = q_{\text{max}} = \sqrt{p_i \cdot p_j}$. It follows that ordering according to q^2 implies $\xi < z^2$.

The region $\xi > z^2$ is therefore a ‘dead zone’ for the shower evolution. In such a zone the physical radiation is not logarithmically enhanced, but not completely absent as happens in the standard algorithm. We therefore need to improve the HERWIG parton showers by the use of matrix-element corrections.

MATRIX-ELEMENT CORRECTIONS

According to [7], we populate the ‘dead zone’ of the phase space using the exact $\mathcal{O}(\alpha_S)$ matrix element (hard correction). We also correct the emission in the already-populated region using the first-order result any time an emission is capable of being the ‘hardest so far’ (soft correction), where the hardness of an emission is measured in terms of the transverse momentum of the emitted parton relative to the splitting one.

We consider the process $q(p_1)\bar{q}'(p_2) \rightarrow V(q)g(p_3)$, define the Mandelstam variables $\hat{s} = (p_1 + p_2)^2$, $\hat{t} = (p_1 - p_3)^2$ and $\hat{u} = (p_2 - p_3)^2$ and obtain the total phase-space limits

$$m_V^2 < \hat{s} < s, \quad (3)$$

$$m_V^2 - \hat{s} < \hat{t} < 0, \quad (4)$$

s being the total centre-of-mass energy. We observe that the soft singularity corresponds to $s = m_V^2$ and the lines $\hat{t} = 0$ and $\hat{t} = m_V^2 - \hat{s}$ to collinear gluon emission.

After relating the parton shower variables z and ξ to \hat{s} and \hat{t} , as done in [6], and setting $\xi < z^2$, one can get the HERWIG phase-space limits in terms of \hat{s} and \hat{t} . In Fig. 1 we plot the total and the HERWIG phase space for $\sqrt{s} = 200$ GeV and $m_V = 80$ GeV; the soft and collinear singularity are inside the HERWIG region.

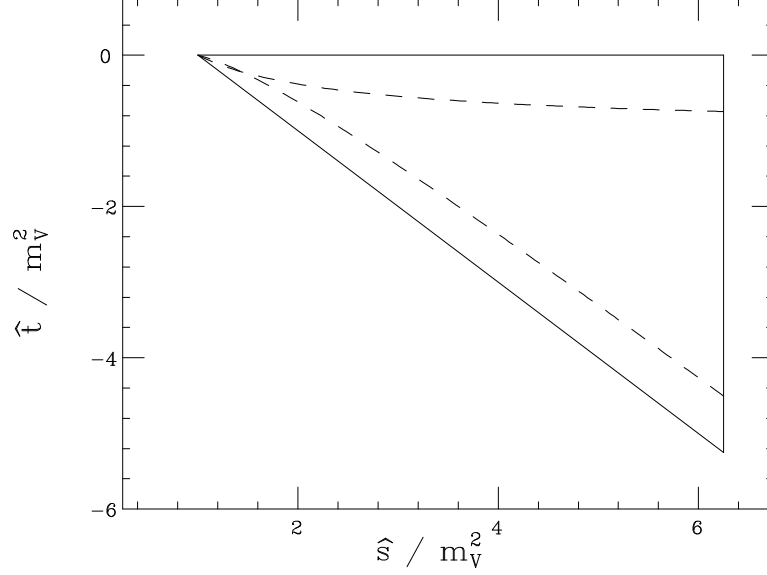


FIGURE 1. Total (solid line) and HERWIG (dashed) phase space limits for $\sqrt{s} = 200$ GeV and $m_V = 80$ GeV.

We calculate the differential cross section with respect to \hat{s} and \hat{t} following the prescription of [18], where it is shown that, assuming that the rapidity and the virtuality of the vector boson are fixed by the Born process, the following factorization formula holds:

$$d^2\sigma = \sigma_0 \frac{f_{q/1}(\chi_1) f_{\bar{q}'/2}(\chi_2)}{f_{q/1}(\eta_1) f_{\bar{q}'/2}(\eta_2)} \frac{C_F \alpha_S}{2\pi} \frac{d\hat{s} d\hat{t}}{\hat{s}^2 \hat{t} \hat{u}} \left[(m_V^2 - \hat{u})^2 + (m_V^2 - \hat{t})^2 \right], \quad (5)$$

where $f_{q/1}(\chi_1)$ and $f_{\bar{q}'/2}(\chi_2)$ are the parton distribution functions of the scattering partons inside the incoming hadrons 1 and 2 for energy fractions χ_1 and χ_2 in the process $q\bar{q}' \rightarrow Vg$, while $f_{q/1}(\eta_1)$ and $f_{\bar{q}'/2}(\eta_2)$ refer to the Born process.

A similar treatment holds for the Compton process $q(p_1)g(p_3) \rightarrow q'(p_2)V(q)$, as discussed in [6].

The distribution (5) or the equivalent one for the Compton process is implemented to generate events in the missing phase space and in the populated region every time an emission is the hardest so far.

TRANSVERSE MOMENTUM DISTRIBUTIONS: HERWIG RESULTS

An interesting observable to study is the vector boson transverse momentum, which is constrained to be $q_T < m_V$ in the soft/collinear approximation. After matrix-element corrections, a fraction of events at higher q_T is to be expected. In Figs. 2 and 3 we plot the W q_T distribution at the Tevatron and at the LHC,

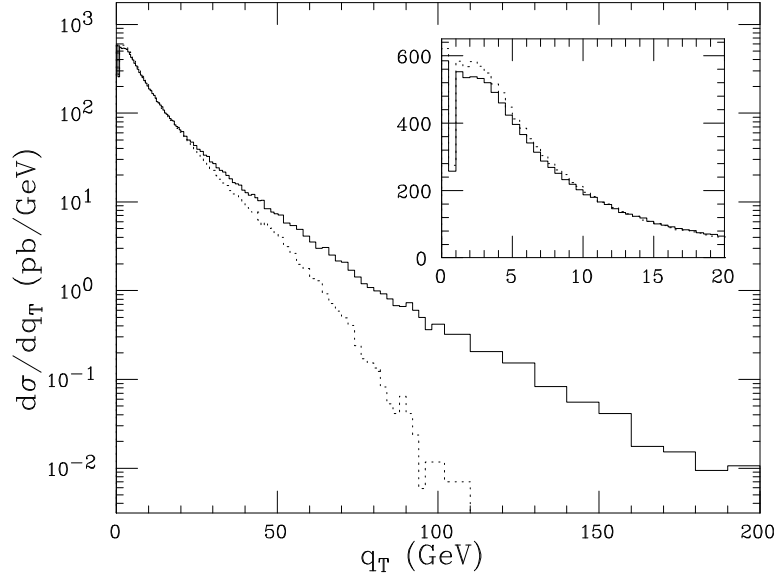


FIGURE 2. W transverse momentum distributions at the Tevatron, according to HERWIG 6.1 (solid line) and HERWIG 5.9 (dotted).

according to HERWIG 5.9 and HERWIG 6.1, the new version including matrix-element corrections, for $q_{T\text{int}} = 0$. We see a big effect at large q_T : after some q_T the 5.9 version does not generate events anymore, while we still have a non-zero cross section after matrix-element corrections.

Moreover, a slight suppression can be seen at small q_T . It is related to the fact that, although we are providing the shower with the tree-level $\mathcal{O}(\alpha_S)$ corrections, virtual contributions are missing and we still get the leading-order cross section. The enhancement at large q_T is therefore compensated by a suppression in the low- q_T range.

It is now interesting to compare the HERWIG results with some Tevatron data. In Fig. 4 we compare the HERWIG 6.1 distribution with some $D\bar{O}$ data [19] and find reasonable agreement over the whole q_T range. As shown in [6], smearing the HERWIG curve to account for detector effects must be included to achieve this agreement. Also, we do not see any relevant impact of setting $q_{T\text{int}} = 1$ GeV after detector corrections. In Fig. 5 we compare the HERWIG 5.9 and 6.1 results, for different values of $q_{T\text{int}}$, with some CDF data [20], already corrected for detector effects. We find good agreement after matrix-element corrections, while the 5.9 version is not able to fit in with the data for $q_T > 50$ GeV. At low q_T , the best agreement to the data is obtained for $q_{T\text{int}} = 2$ GeV, as shown in Fig. 6. While the Z distribution strongly depends on $q_{T\text{int}}$ at small q_T , in [21] and Fig. 7 it is shown that the ratio R of the W and Z spectra is approximately independent of it.³ Such a ratio is one of the main inputs for the W mass measurement in hadron

³) The negative slopes of the plots in Fig. 7 are due to the W/Z mass difference.

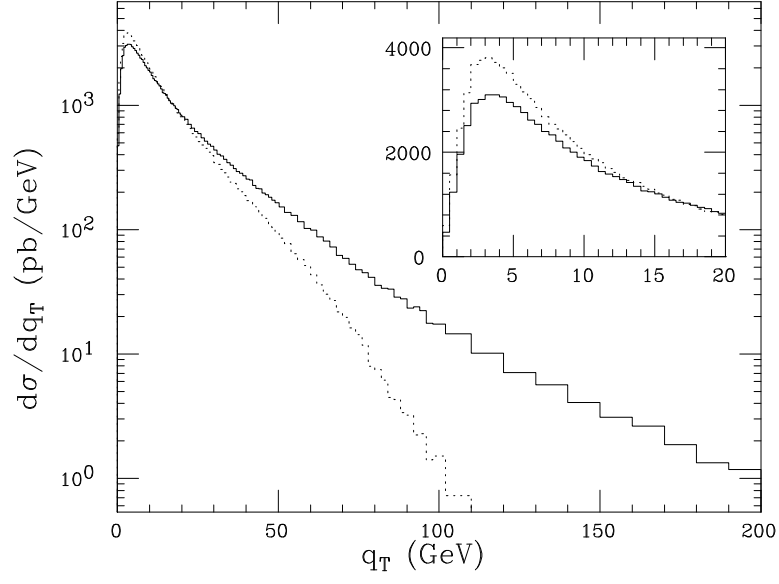


FIGURE 3. As in Fig. 2, but at the LHC.

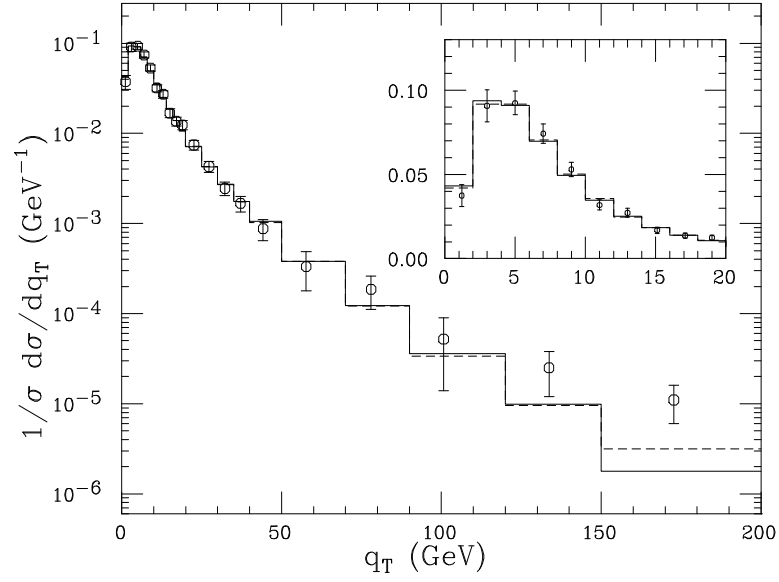


FIGURE 4. Comparison of the DØ data to the HERWIG 6.1 result, for an intrinsic q_T of 0 (solid line) and 1 GeV (dashed).

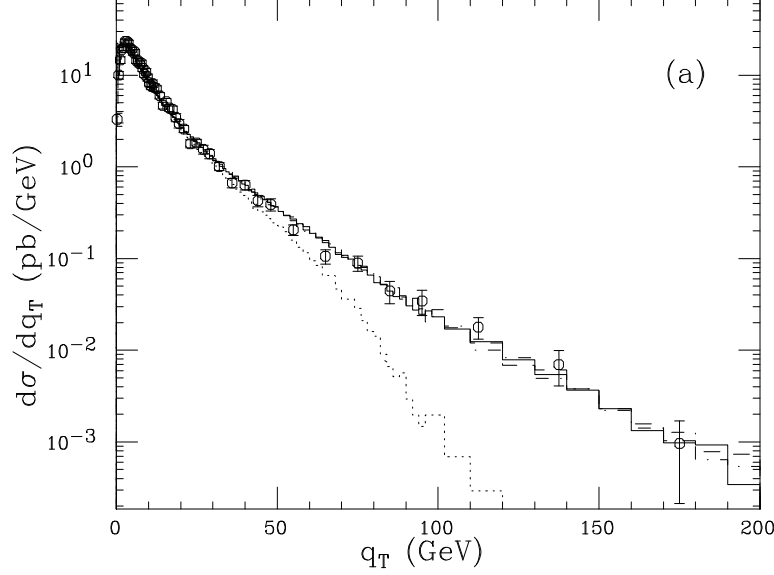


FIGURE 5. Comparison of the CDF data on the Z transverse momentum to HERWIG 5.9 (dotted line) and 6.1 for an intrinsic q_T of 0 (solid line), 1 GeV (dashed) and 2 GeV (dot-dashed).

collisions and it is good news that it does not depend on unknown non-perturbative parameters.

RESUMMED CALCULATIONS

Another possible approach to study the vector boson transverse momentum distribution consists of resumming the logarithmic terms $l = \log(m_V^2/q_T^2)$ in the low- q_T range. It is interesting to compare the HERWIG phenomenological results with those of some resummed calculations, in particular [13] and [14]. According to [9], the resummed differential cross section for W production can be written as:

$$\begin{aligned} \frac{d^2\sigma}{dm_V^2 dq_T^2} &= \sigma_0 \frac{d}{dq_T^2} \sum_{q,q'} |V_{q\bar{q}'}|^2 \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) \\ &\times \left[f_{q/1}(x_1, q_T) f_{\bar{q}'/2}(x_2, q_T) \exp[S(m_V, q_T)] + (q \leftrightarrow \bar{q}') \right], \end{aligned} \quad (6)$$

where $V_{q\bar{q}'}$ is the relevant Cabibbo–Kobayashi–Maskawa matrix element and $\tau = m_V^2/s$. In (6), $\exp[S(m_V, q_T)]$ is a Sudakov-like form factor which resums the large logarithms associated to the initial-state radiation. It reads:

$$S(m_V, q_T) = - \int_{q_T^2}^{m_V^2} \frac{d\mu^2}{\mu^2} \left[A(\alpha_S(\mu^2)) \log \frac{m_V^2}{\mu^2} + B(\alpha_S(\mu^2)) \right], \quad (7)$$

where $A(\alpha_S)$ and $B(\alpha_S)$ can be expanded as:

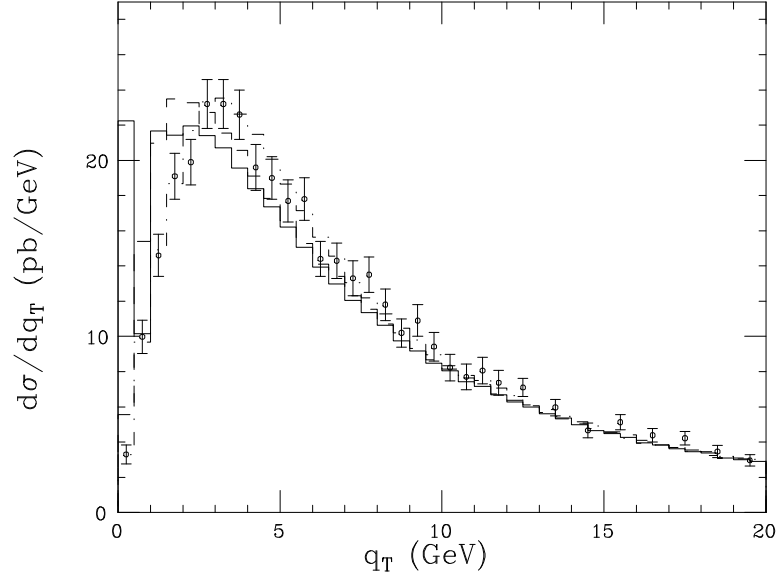


FIGURE 6. As in Fig. 5, but in the low- q_T range.

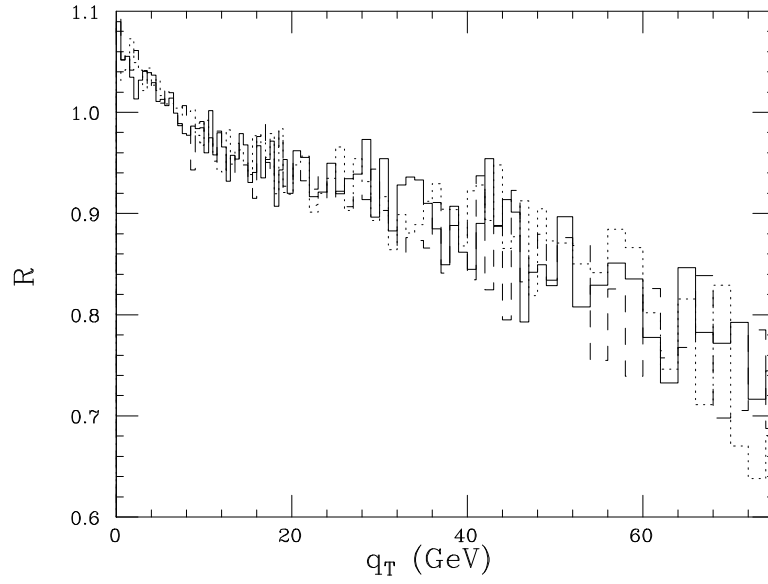


FIGURE 7. The ratio of the W to the Z spectrum at small q_T for an intrinsic transverse momentum of zero (solid), 1 (dashed) and 2 GeV (dotted).

$$A(\alpha_S) = A_1\alpha_S + A_2\alpha_S^2 + \dots ; B(\alpha_S) = B_1\alpha_S + B_2\alpha_S^2 + \dots \quad (8)$$

As far as the logarithms which contribute to the resummation are concerned, two conflicting nomenclatures exist. One consists of looking at Sudakov exponent, where the leading logarithms (LL) are $\sim \alpha_S^n l^{n+1}$ and the next-to-leading ones (NLL) $\sim \alpha_S^n l^n$. It is straightforward to show that the LL contributions are obtained by keeping only the A_1 term in the expansions (8) while NLL accuracy is achieved by considering A_2 and B_1 as well. In this sense, the approach [14] is NLL.

Another classification relies on the expansion of the exponent

$$S(m_V, q_T) = \sum_n c_{n,n+1} \alpha_S^n l^{n+1} + \sum_n c_{n,n} \alpha_S^n l^n, \quad (9)$$

where the leading term is $\sim \alpha_S l^2$, so that the leading contributions to $\exp[S(m_V, q_T)]$ are $\sim \alpha_S^n l^{2n}$, terms $\sim \alpha_S^n l^{2n-1}$ being next-to-leading. This is equivalent to saying that in the differential cross section the LL and NLL contributions are $\sim (1/q_T^2) \alpha_S^n l^{2n-1}$ and $\sim (1/q_T^2) \alpha_S^n l^{2n-2}$ respectively. According to this nomenclature, the calculations [13] and [15] are NNLL and NNNLL respectively.

In the b -space formalism, following [14], the differential cross section reads:

$$\begin{aligned} \frac{d^2\sigma}{dm_V^2 dq_T^2} &= \frac{\sigma_0}{4\pi} \sum_{q,q'} |V_{q\bar{q}'}|^2 \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) \int d^2b e^{iq_T \cdot \vec{b}} \\ &\times \left[f_{q/1}(x_1, c_1/b) f_{\bar{q}'/2}(x_2, c_1/b) \exp[S(m_V, b)] + (q \leftrightarrow \bar{q}') \right], \end{aligned} \quad (10)$$

where c_1 and c_2 are integration constants of order 1 and $S(m_V, b)$ is the Sudakov exponent in b -space.

For high b values, i.e. small q_T , non-perturbative effects are taken into account via a Gaussian function $F_{NP} = \exp(-g b^2)$, as suggested in [11]. In both [13] and [14] the value $g = 3 \text{ GeV}^2$ is chosen.

Also, in order to allow resummed calculations to be reliable even at large q_T , we wish to match the calculations of [13] and [14] to the exact $\mathcal{O}(\alpha_S)$ result. We add the first-order cross section to the resummed result and, in order to avoid double counting, we subtract off the term which they have in common, which is the $q_T \rightarrow 0$ limit of the exact $\mathcal{O}(\alpha_S)$ result. According to our prescription, the matching works fine if at the point $q_T = m_V$ we have a continuous distribution.

COMPARISON OF HERWIG AND RESUMMED CALCULATIONS

A detailed and general discussion on the comparison of angular-ordered parton shower algorithms with resummed calculations for Drell–Yan processes was already performed in [22], where the authors showed that, for $\tau \rightarrow 1$, HERWIG always accounts for the term A_1 , corresponding to the leading logarithms in the exponent, and B_1 as well. Furthermore, one is able to account for the NLL term A_2 by

simply modifying the Altarelli–Parisi splitting function introducing a second-order contribution

$$P'_{qq}(\alpha_S, z) = \frac{\alpha_S}{2\pi} C_F \frac{1+z^2}{1-z} + \frac{C_F}{2} \left(\frac{\alpha_S}{\pi} \right)^2 \frac{K}{1-z}, \quad (11)$$

where the K factor is given by:

$$K = C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f, \quad (12)$$

N_f being the number of flavours, $C_F = 4/3$ and $C_A = 1/2$. This is equivalent to redefining the QCD parameter Λ to the ‘Monte Carlo’ Λ_{MC} :

$$\Lambda_{MC} = \Lambda \exp(K/4\pi\beta_0), \quad (13)$$

with $\beta_0 = (11C_A - 2N_f)/(12\pi)$. Even after these replacements, the HERWIG algorithm cannot be considered completely accurate at the next-to-leading level, since it is still missing higher-order contributions in the strong coupling constant or the parton distribution functions (see, for instance, the discussion in [3]).

In Fig. 8 we show the W transverse momentum distribution at the Tevatron in the low- q_T range according to HERWIG 6.1 and the calculations of [13] in q_T -space and of [14] in q_T - and b -space. The HERWIG curve lies within the range of the resummed calculations, which is a reasonable result, considering that we are actually comparing different approaches. In Fig. 9 we consider the whole q_T range, with the resummations matched to the exact first-order amplitude. We find that the matching works fine only for the approach [14] in q_T -space, the others showing a step at $q_T = m_W$, due to uncompensated contributions of order α_S^2 or higher. The well-matched distribution agrees with the HERWIG 6.1 prediction at large q_T .

CONCLUSIONS

We studied the initial-state radiation in vector boson production according to the HERWIG event generator and some resummed calculations. In particular, we investigated the effect of the recently-implemented matrix-element corrections to the HERWIG algorithm. We found a big effect of such corrections on W/Z transverse momentum distributions at the Tevatron and at the LHC, and good agreement with the DØ and CDF data, with a crucial role played by such corrections in order to be able to fit in with the data at large q_T . We also found that, even though the spectra at small q_T do depend on the intrinsic non-perturbative transverse momentum, the ratio of the W to the Z spectrum is roughly independent of it. We then considered some resummed calculations, which we matched to the exact $\mathcal{O}(\alpha_S)$ matrix element, which makes them reliable at large q_T as well. We found reasonable agreement of such approaches with HERWIG and fine matching only

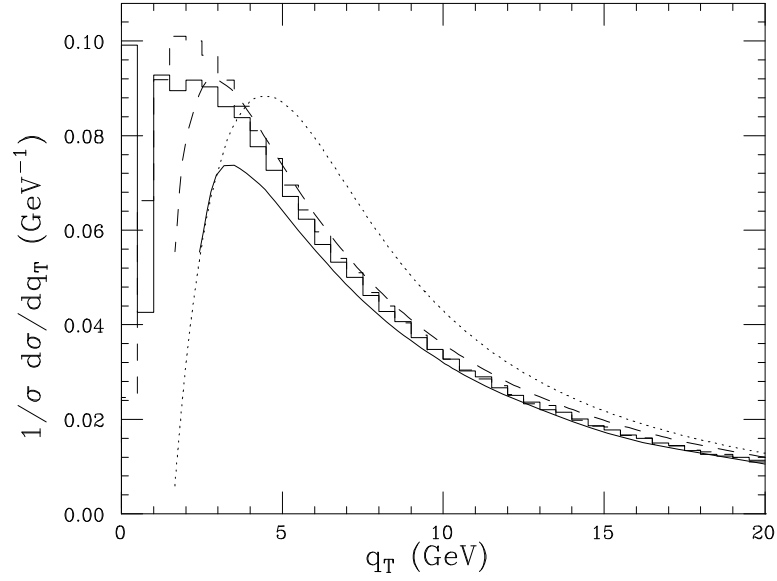


FIGURE 8. The W q_T distribution at the Tevatron, according to HERWIG with matrix-element corrections, with zero intrinsic q_T (solid histogram) and an $q_{T,\text{int}}$ of 1 GeV (dashed histogram), compared with the resummed results of [14] in q_T -space (solid line) and in b -space (dotted line) and of [13] (dashed line) in q_T -space.

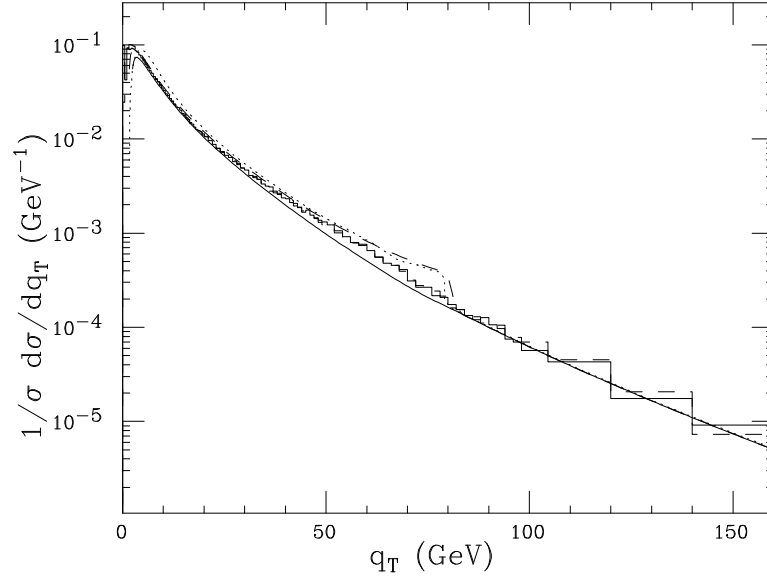


FIGURE 9. As in Fig. 8, but with the resummed results matched with the exact $\mathcal{O}(\alpha_S)$ matrix element.

for the calculation which keeps all the next-to-leading logarithms in the Sudakov exponent in q_T -space.

Finally, we have to say that the discussed method of improving the initial-state radiation in parton-shower Monte Carlo simulations can be extended to a wide range of interesting processes for the phenomenology of hadron colliders. The implementation of hard and soft corrections to top and Higgs production is in progress.

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